



**WORKING PAPER**

**N° 2022-14**

**STRATEGIC DEBT IN A MIXED DUOPOLY:  
THE LIMITED LIABILITY EFFECT**

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TEPP – Theory and Evaluation of Public Policies - FR CNRS 2042

# Strategic debt in a mixed duopoly: The limited liability effect

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June 18, 2022

## Abstract

We study the impact of the private firm's debt on the equilibrium of a mixed duopoly by focusing on the effect of limited liability. The debt, combined with the limited liability clause, encourages the private firm to take into account only those states of the nature where demand is high. Debt therefore drives the private firm to increase its production. In response, the public firm reduces its production. Total production is increasing, causing the equilibrium price to fall and the consumer surplus to rise. The social welfare increases thanks to a more efficient allocation of total production between the two firms.

Keywords: Mixed duopoly, strategic debt, privatization.

JEL Classification numbers: D43, L13, L32.

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# 1 Introduction

In this study, we analyze the level of debt chosen by a private firm in a mixed duopoly and its impact on its production strategy, as well as on the social welfare.

Interactions between the financial structure of firms and their product market strategies have been the subject of numerous studies.<sup>1</sup> These studies focused on the case where all firms are private. There are, however, many industries in which public or partly public firms are present, particularly in Europe and Asia.<sup>2</sup> It therefore seems interesting to extend the existing literature on the interactions between financing and production strategies to industries where private firms compete with public firms. This work was initiated by Jacques (2021). This study continues it by looking at another mechanism through which the debt of the private firm has an impact on the equilibrium obtained in a mixed duopoly. Jacques (2021) focused on the impact of the private firm's debt on the state-owned firm's production strategy. This study neutralizes this effect and focuses on the effect of the private firm's debt on its production strategy due to its limited liability.

The literature has highlighted several reasons why a firm's financial structure has an impact on its strategy. The first, studied in particular by Brander and Lewis (1986), is due to the limited liability of firms. If a firm goes bankrupt, its shareholders are not liable for the residual debts after the liquidation of the firm. With limited liability, shareholders' earnings are at least zero, even if the profits of the firm they own become negative. If firms make their production decisions before they know the state of demand or the level of costs, this leads them to neglect the states of the nature where the profits are negative and to over-weight the states of the nature where they are positive. If firms compete in quantities,<sup>3</sup> debt drives firms to produce more. As quantities are generally strategic substitutes, competing firms are then encouraged to reduce production. In this type of modelling, debt has a strategic value. The second effect is related to bankruptcy costs. A bankruptcy generates costs for the managers of a firm. Its resolution requires administrative or judicial proceedings, which have costs in money and time. Bankruptcy can also damage the reputation of managers and their future prospects.<sup>4</sup> Brander and Lewis (1988) studied the impact of bankruptcy costs.<sup>5</sup> The existence of bankruptcy costs encourages firms to over-weight the states of nature around the threshold of bankruptcy. Altering the firm's output may increase the firm's earnings sufficiently in some of these states to avoid bankruptcy. An indebted firm therefore modifies its production choices to reduce its probability of bankruptcy and to reduce the expectation of costs caused by bankruptcy. As the overweighted states change with the level of debt, the authors obtain a nonmonotonous relationship between the level of debt and the

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<sup>1</sup>Faure Grimaud (1998), Cestone (1999) and Jacques (2022, chapter 15) provide surveys of this literature.

<sup>2</sup>Examples include aeronautics, car production, banks, air transport, electricity generation, etc.

<sup>3</sup>Showalter (1995) investigated the case of price competition with differentiated goods. The results then depend on the nature of the uncertainty. If the uncertainty is about demand, an indebted firm increases its price and its competitor reacts by increasing its price. On the other hand, if the uncertainty relates to the costs of production, the debt encourages the firm to lower its price and its competitor imitates it.

<sup>4</sup>They may find it difficult in the future to find a job as a manager or to obtain funds to create a new business.

<sup>5</sup>Brander and Lewis (1986) chose to ignore them to isolate the effects of limited liability.

level of production. Bolton and Scharfstein (1990) emphasized a third effect of debt. A firm's debt can have a direct effect on the production strategy of competing firms. The latter may seek to reduce the firm's profits in debt to increase its likelihood of bankruptcy and eliminate it in order to reduce future competition.

Jacques (2021) looked at the impact of bankruptcy costs in a mixed duopoly. His contribution is different from that of Brander and Lewis (1988). The debt has no direct effect on the production strategy of the private firm. To neutralize the effects present in Brander and Lewis (1986, 1988), the author modified the chronology of the game. Firm production choices are made after firms have observed the true level of demand. The effect of indebtedness does not therefore consist in modifying the weightings placed on the various states of nature by the private firm when it chooses its level of production. But, the debt of the private firm can change the quantity chosen by the public firm. Indeed, in a mixed duopoly, it is generally assumed that the objective of the public firm is to maximize the social welfare. The costs of bankruptcy are reducing the level of this surplus. The public firm may therefore have an interest in reducing its production if this reduction prevents the private firm from going bankrupt. The reduction in the output of the public firm, however, causes a fall in the consumer surplus and increases the gap between the total output of the industry and the socially optimal level of output. The public firm chooses its level of production by cutting between these two evils: the bankruptcy of the private firm and the reduction of the consumer surplus. If the costs of bankruptcy are high and the reduction of its production necessary to save the private firm from bankruptcy is low, the state-owned firm chooses to save the private firm from bankruptcy. In the opposite case, the public firm favours the interests of consumers and lets the private firm go bankrupt. The private firm anticipates this rule of choice and chooses the highest level of debt, which encourages the public firm to avoid bankruptcy if demand is low. The bankruptcy of the private firm never happens on the equilibrium path. If the costs of bankruptcy are not too high, the use of debt allows an increase in the social welfare. On the other hand, if bankruptcy costs are high, the use of debt reduces the social surplus. To understand the intuition behind these results, it is necessary to return to the classical results of the Cournot competition in a mixed oligopoly. This detour will also provide a better understanding of the results obtained in this study.

In a mixed oligopoly, the state-owned firm seeks to maximize the social welfare. When competition is in quantities,<sup>6</sup> the public firm chooses the level of production which equalizes its marginal cost with the equilibrium price. It therefore produces more than a private firm. In response, competing private firms reduce their production. Total production is higher than in a private oligopoly, which contributes to increasing the social surplus. However, the allocation of production between firms is not the one that minimizes total production costs. If all firms have the same strictly convex cost function, in equilibrium, the marginal cost of the public firm is higher than that of private firms. This inefficiency in the allocation of production between firms tends to reduce the social welfare.

By taking this Cournot equilibrium as a reference point, a commitment by a private firm to slightly

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<sup>6</sup>De Fraja and Delbono (1989) were among the first to study the properties of this type of competition in a mixed oligopoly. For surveys of the literature on mixed oligopolies, see De Fraja and Delbono (1990) and Jacques (2022, chapter 13).

increase its production or a commitment by the public firm to reduce its production causes an increase in the social surplus.<sup>7</sup> Indeed, these commitments reduce production inefficiency by slightly modifying the allocation of production between firms. When it is the public firm that commits to reduce its production, total production decreases, which contributes to a reduction in the social welfare. But if the output variations are small, the first effect dominates and the social welfare increases. This is what happens in Jacques (2021) when bankruptcy costs are low. The strategic indebtedness of the private firm pushes the public firm to slightly reduce its production and in reaction the private firm slightly increases its own. When bankruptcy costs are high, the output of the state-owned firm falls more sharply and the effect of the fall in total output dominates the positive effect of a better distribution of output among firms.

In this study, we get similar results, but the mechanism is different. In Jacques (2021), debt causes a change in the best response of the public firm to the quantity produced by the private firm. The public firm reduces its production to reduce the risk of bankruptcy of the private firm. In this study, this mechanism is neutralized by assuming that the costs of bankruptcy are zero. At the same time, the chronology of the game is changed and it is assumed that firms determine their production levels before observing the true level of demand. The limited liability effect, studied by Brander and Lewis (1986) in a private duopoly, is reintroduced. Unlike Jacques (2021), the debt in this study changes the production strategy of the private firm, not that of the public firm. Clearly separating the two effects in different studies, as Brander and Lewis (1986, 1988) did, provides a good understanding of the mechanisms that contribute to firms' debt having an impact on their strategies in the market for goods in imperfect competition.

If the private firm incurs enough debt to go bankrupt when demand is low, it modifies the weightings it assigns to the different possible states of demand when choosing production. This encourages it to increase its production. As the quantities chosen by the two firms are strategic substitutes, the public firm reacts by reducing its production. Total production is increasing, causing the equilibrium price to fall and the consumer surplus to rise. The impact on the social welfare is a priori more ambiguous. Neglecting the states of nature in which demand is low could induce the private firm to choose a level of production that exceeds the socially optimal level and could lead to a reduction in the social surplus. However, this never happens in

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<sup>7</sup>The literature on mixed oligopolies has identified several mechanisms to make these commitments credible.

A first way is to modify the timeline of the game. The private firm prefers to play first to commit to a higher level of production. The public firm often prefers to play second to let the private firm choose a high level of production by implicitly threatening to produce more if the private firm produces less. The impact of the game timeline was highlighted by Harris et Wiens (1980). Pal (1998) was the first to show that, if firms may choose when they set their level of production, a sequential timeline emerges in equilibrium. Both firms prefer that the private firm assume the leadership role. But the chronology in which the state-owned firm is the leader is also often an equilibrium (Pal, 1998; Jacques, 2004).

A second way to commit on different levels of production is to delegate the management of firms to managers and assign them objective functions different from profit maximization (for the private firm) or welfare maximization (for the state-owned firm). The manipulation of objectives assigned to managers has been studied, in particular, by Barros (1995), White (2001, 2002), Benassi, Chirco and Scrimatore (2014) and Ouattara (2013). A way to change the weightings of various components of the objective function of the public firm consists in partially privatizing it. By varying the degree of private shareholding in the capital of the public firm, one modifies the weighting given to the firm's profit in the objective function (Matsumura, 1998).

A third way is to distort firms' choices of capacities (Nishimori and Ogawa, 2004; Lu and Poddar, 2005; Meunier, 2008). The state-owned firm underinvests in capacity to increase its marginal cost and commit itself to reduce its production. The private firm adopts an opposite strategy.

Jacques (2021) and this study present a fourth means, which is based on the debt of the private firm

equilibrium, in this model. The private firm chooses to resort to debt in a strategic way only if the distortion that this induces on its choice of production is limited. The use of strategic debt always causes an increase in the profit of the private firm. This first result differs from the results obtained by Brander and Lewis (1986) in a private duopoly. In Brander and Lewis (1986), the use of debt allows a firm to increase its expectation of profit for a given debt of the other firm, but this is at the expense of the competing firm. As the two firms choose to go into debt, in equilibrium, profits are lower than without debt. The second result (the increase in the social surplus) differs from that obtained by Jacques (2021) where there are values of the parameters for which the social surplus can decrease.

It is interesting to compare the results obtained in a mixed duopoly with those found in a private duopoly. It can be seen that the use of debt for strategic reasons is a more common phenomenon in a mixed duopoly than in a private duopoly, in the sense that the zone of values of the parameters for which firms use strategic debt in the private duopoly is included in the zone where strategic debt is used by the private firm in a mixed duopoly. This does not necessarily imply that a private firm is more indebted in a mixed duopoly than in a private duopoly. Indeed, the profits of the private firm in the mixed duopoly (without debt) are lower than in a private duopoly. The level of debt from which the debt has an impact on the production level of firms is therefore lower in a mixed duopoly than in a private duopoly. We could therefore observe higher levels of debt in a private duopoly, but which have no strategic impact and a lower level of debt for the private firm in the mixed duopoly, but which changes the production strategy of this firm. We also compare the values of the social welfare in the two types of duopoly to determine whether it is desirable to privatize the public firm. In this model, unlike the results of Jacques (2021), privatization is never desirable.

The organization of the study is as follows. The model assumptions are presented in the next section (section 2). Section 3 analyses the effects of private firm debt in a mixed duopoly and determines the financial structure chosen by the private firm. We then repeat the exercise, in section 4, for the private duopoly. Section 5 compares the social welfares obtained in the two types of duopoly to determine whether there are cases in which privatization of the public firm would increase the social surplus. Section 6 concludes with a summary of the main findings before outlining future avenues for research.

## 2 Model

We are studying a duopoly in which two firms compete in quantities with homogeneous goods. One of the firms (firm 1) is state-owned and has the objective of maximizing the social welfare. The other firm (firm 2) is private and strives to maximize its profit. The cost functions of the two firms are identical and equal to<sup>8</sup>  $c(q_i) = q_i^2$ .

The level of demand is uncertain. The inverse demand function is linear:  $p = \max(0, \alpha - q_1 - q_2)$ .  $\alpha$  is a

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<sup>8</sup>Using quadratic cost functions makes it possible to obtain that firms have different marginal costs in equilibrium, without having to assume that the private firm is more efficient than the public firm.

random variable that can take two values:  $\alpha_L$  with probability  $\mu$  and  $\alpha_H$  with probability  $1 - \mu$ .  $\alpha_H > \alpha_L$ . We note  $\bar{\alpha}$  the expected value of  $\alpha$ . Limiting the states of nature to two makes it possible to obtain analytical expressions for the different variables. Firms learn the true value of  $\alpha$  only after have chosen the quantities they wish to produce, so the timing is different from that adopted in Jacques (2021).<sup>9</sup>

Before the firms compete in quantities, the private firm can go into debt in a perfectly competitive financial market whose interest rate is normalized to 0. This debt has only a strategic objective. The funds raised are immediately distributed to the shareholders of the private firm. The private firm must then repay an amount of  $D$  at the end of the Cournot competition stage. If the firm is unable to repay that amount, it goes bankrupt. The shareholders of the private firm benefit from a limited liability clause. To focus on the effects of limited liability, no bankruptcy costs are introduced into this model.

We assume that firms, their creditors and consumers are risk neutral.

The timeline of the game is as follows: (1) The private firm chooses the level of its debt. (2) Firms choose their production levels. (3) The true value of  $\alpha$  is observed, the equilibrium price is determined so as to equalize the supply and demand and the gains are distributed.

In section 5, a preliminary step is introduced in which the State decides to keep the public firm or privatise it.<sup>10</sup>

We note  $q_i$  the quantity produced by firm  $i$ ,  $\pi_i$  the profit of firm  $i$ ,  $p$  the equilibrium price,  $CS$  the consumers surplus,  $W$  the social welfare,  $VD_i$  the value of the debt of firm  $i$  and  $V_i$  the total value of firm  $i$  ( $V_i = \pi_i + VD_i$ ).

### 3 Debt effects in a mixed duopoly

To identify the effects of debt on Cournot competition in a mixed duopoly, we compare the equilibria obtained without and with debt.

#### 3.1 Cournot competition without debt

In the usual way, we calculate the best reply of each firm before determining the quantities chosen in equilibrium and the payoffs obtained.

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<sup>9</sup>Jacques (2001) studies the effect of bankruptcy costs on firm strategy. To be able to focus on this effect, he chose to neutralize the limited liability effect by assuming that firms can observe the value of  $\alpha$  before choosing their level of production (but after choosing their level of debt).

<sup>10</sup>The possibility of partial privatization is not considered unlike Matsumura (1998).

**Firms' best replies:** The private firm seeks to maximize its profit:

$$\begin{aligned}\pi_2(q_1, q_2) &= \mu(\alpha_L - q_1 - q_2)q_2 + (1 - \mu)(\alpha_H - q_1 - q_2)q_2 - q_2^2 \\ &= (\bar{\alpha} - q_1 - q_2)q_2 - q_2^2\end{aligned}$$

Its best response to the quantity produced by the competing firm is given by:

$$\frac{\partial \pi_2}{\partial q_2}(q_1, q_2) = 0 \Leftrightarrow q_2(q_1) = \frac{1}{4}(\bar{\alpha} - q_1)$$

The objective of the public firm is to maximize the social welfare, which is equal to the sum of firms' profits and consumer surplus:

$$W(q_1, q_2) = \frac{1}{2}(q_1 + q_2)^2 + (\bar{\alpha} - q_1 - q_2)q_1 - q_1^2 + (\bar{\alpha} - q_1 - q_2)q_2 - q_2^2$$

The best response of the state-owned firm to the quantity produced by the private firm is determined by:

$$\frac{\partial W}{\partial q_1}(q_1, q_2) = 0 \Leftrightarrow q_1(q_2) = \frac{1}{3}(\bar{\alpha} - q_2)$$

**Quantity equilibrium in the mixed duopoly:** We easily deduce the quantities of the Cournot equilibrium from the best replies of the firms:

$$\left\{ \begin{array}{l} q_1 = \frac{1}{3}(\bar{\alpha} - q_2) \\ q_2 = \frac{1}{4}(\bar{\alpha} - q_1) \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} q_2 = \frac{2}{11}\bar{\alpha} \\ q_1 = \frac{3}{11}\bar{\alpha} \end{array} \right\}$$

Although both firms have the same cost function, the state-owned firm produces more than the private firm in equilibrium, because it takes into account the impact of an increase in its production on the consumer surplus.

**Payoffs for a given  $\alpha$ :** We compute the equilibrium price and the various payoffs according to the value of  $\alpha$ .

$$\begin{aligned}p &= \alpha - \frac{5}{11}\bar{\alpha} & \pi_1 &= \left(\alpha - \frac{8}{11}\bar{\alpha}\right) \frac{3}{11}\bar{\alpha} & \pi_2 &= \left(\alpha - \frac{7}{11}\bar{\alpha}\right) \frac{2}{11}\bar{\alpha} \\ CS &= \frac{25}{242}\bar{\alpha}^2 & W &= \frac{1}{11}\left(5\alpha - \frac{51}{22}\bar{\alpha}\right)\end{aligned}$$

We want the profit of the private firm to be positive even if the demand is low. To ensure that this condition is met, we limit ourselves to the values of the parameters that verify:  $\alpha_L > \frac{7(1-\mu)}{11-7\mu}\alpha_H$ . If this condition is not verified, the best response function of the private firm is different since, due to its limited liability, it would choose its production focusing only on the state of demand where  $\alpha = \alpha_H$ . The debt of the private firm would no longer have an impact on the firm's behavior. In order to focus on the most interesting case, we assume:  $\alpha_L > \frac{7(1-\mu)}{11-7\mu}\alpha_H$ .

It may be noted that the linear specification of the demand function has the effect that the surplus of consumers is independent of  $\alpha$ .



**Expected values of payoffs:** From the previous expressions, we can calculate the expected values of payoffs (before the value of *alpha* is observed):

$$E(\pi_1) = \frac{9}{121}\bar{\alpha}^2 \ ; \ E(\pi_2) = \frac{8}{121}\bar{\alpha}^2 \ ; \ E(SC) = \frac{25}{242}\bar{\alpha}^2 \ ; \ E(W) = \frac{59}{242}\bar{\alpha}^2$$

### 3.2 Cournot competition with debt

It is assumed that the private firm is in debt and must repay  $D$  at the end of the period. First, we consider the value of  $D$  as given. We must distinguish three cases.

If  $D$  is low (lower than  $(\alpha_L - \frac{7}{11}\bar{\alpha})\frac{2}{11}\bar{\alpha}$ ), the private firm is able to repay its debt even if the level of demand is low. The private firm continues to choose its level of production taking into account both states of demand. Its behavior is not affected by its debt. The equilibrium quantities are the same as in the absence of debt.

If  $D$  is very high,<sup>11</sup> the private firm is not able to repay its debt even if the demand is high. The private firm goes bankrupt and gets a zero profit in all states of the demand no matter how much it chooses to produce. The optimal behavior of this firm is then not defined. This case therefore poses a problem of resolution and does not offer much interest. So we choose to ignore it by imposing that the value of  $D$  is never higher than the profit of the private firm when the demand is high.

The interesting case is when  $D$  lies between the two previous cases. The firm is not able to pay down its debt when the demand is low, but it can do so when the demand is high. As the firm has limited liability, it is concerned only with the state of the nature in which the demand is high. It will therefore choose the quantity that maximizes its profit in this state of nature. Debt therefore has an impact on the best response function of the private firm. We focus on this case in the rest of this section.

**Best reply of the private firm:** Private firm seeks to maximize profit:

$$\pi_2(q_1, q_2) = (1 - \mu) [(\alpha_H - q_1 - q_2)q_2 - q_2^2 - D]$$

Its best reply to the quantity produced by the competing firm is given by:

$$\frac{\partial \pi_2}{\partial q_2}(q_1, q_2) = 0 \Leftrightarrow q_2(q_1) = \frac{1}{4}(\alpha_H - q_1)$$

The reaction function of the public firm does not change.<sup>12</sup>

**Equilibrium:** We calculate the selected quantities in equilibrium:

<sup>11</sup>We will specify the limit of this interval a little further.

<sup>12</sup>It would be different if there were bankruptcy costs. See Jacques (2021).

$$\left\{ \begin{array}{l} q_1 = \frac{1}{3}(\bar{\alpha} - q_2) \\ q_2 = \frac{1}{4}(\alpha_H - q_1) \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} q_1 = \frac{4\bar{\alpha} - \alpha_H}{11} \\ q_2 = \frac{3\alpha_H - \bar{\alpha}}{11} \end{array} \right\}$$

The quantity produced by the private firm is higher ( $\frac{3\alpha_H - \bar{\alpha}}{11} > \frac{2}{11}\bar{\alpha}$ ) than without debt. As quantities are strategic substitutes, the public firm reacts by reducing its own quantity ( $\frac{4\bar{\alpha} - \alpha_H}{11} < \frac{3}{11}\bar{\alpha}$ ).

**Payoffs for a given  $\alpha$ :** We calculate the equilibrium price and the payoffs based on the level of demand. We assume that the creditors are domestic investors. So we integrate their earnings expectations into the social surplus.

$$\begin{aligned} p(\alpha) &= \alpha - \frac{3\bar{\alpha} + 2\alpha_H}{11} \\ \pi_1(\alpha) &= \left( \alpha - \frac{7\bar{\alpha} + \alpha_H}{11} \right) \frac{4\bar{\alpha} - \alpha_H}{11} \\ \pi_2(\alpha) &= \begin{cases} 0 & \text{si } \alpha = \alpha_L \\ \left( \alpha_H - \frac{2\bar{\alpha} + 5\alpha_H}{11} \right) \frac{3\alpha_H - \bar{\alpha}}{11} - D & \text{si } \alpha = \alpha_H \end{cases} \\ VD(\alpha) &= \begin{cases} \left( \alpha_L - \frac{2\bar{\alpha} + 5\alpha_H}{11} \right) \frac{3\alpha_H - \bar{\alpha}}{D} & \text{si } \alpha = \alpha_L \\ \frac{3\alpha_H - \bar{\alpha}}{11} & \text{si } \alpha = \alpha_H \end{cases} \\ CS(\alpha) &= \frac{1}{2} \left( \frac{3\bar{\alpha} + 2\alpha_H}{11} \right)^2 \\ W(\alpha) &= \frac{1}{2} \left( \frac{3\bar{\alpha} + 2\alpha_H}{11} \right)^2 + \left( \alpha - \frac{7\bar{\alpha} + \alpha_H}{11} \right) \frac{4\bar{\alpha} - \alpha_H}{11} + \left( \alpha - \frac{2\bar{\alpha} + 5\alpha_H}{11} \right) \frac{3\alpha_H - \bar{\alpha}}{11} \end{aligned}$$

We assumed that we were in the case where  $D$  has an intermediate value. This is the case if  $\pi_2(\alpha_L) < 0 < \pi_2(\alpha_H)$ . The lower end of that range has already been specified. The upper end can now be determined. We must have  $D < \left( \alpha_H - \frac{2\bar{\alpha} + 5\alpha_H}{11} \right) \frac{3\alpha_H - \bar{\alpha}}{11}$ .

**Expected payoffs:** On the basis of the preceding expressions, we compute the expected payoffs.

$$\begin{aligned} E(\pi_1) &= \left( \frac{4\bar{\alpha} - \alpha_H}{11} \right)^2 ; \quad E(\pi_2) = (1 - \mu) \left[ 2 \left( \frac{3\alpha_H - \bar{\alpha}}{11} \right)^2 - D \right] \\ E(VD) &= \mu \left( \alpha_L - \frac{2\bar{\alpha} + 5\alpha_H}{11} \right) \frac{3\alpha_H - \bar{\alpha}}{11} + (1 - \mu) D \\ E(CS) &= \frac{1}{2} \left( \frac{3\bar{\alpha} + 2\alpha_H}{11} \right)^2 ; \quad E(W) = \frac{23\bar{\alpha}^2 + 60\bar{\alpha}\alpha_H - 24\alpha_H^2}{242} \end{aligned}$$

### 3.3 Comparison and choice of the level of debt

When the private firm becomes heavily indebted, it chooses to increase its production. The public company reduces its production in response, but by a lower amount. Indeed, quantities are strategic substitutes and

the slope of the public firm's best response function is less than 1 (in absolute value). The total quantity goes up. The equilibrium price goes down and the consumer surplus raises. The profit of the public firm decreases.<sup>13</sup> The effect on the social welfare and profits of firms is more ambiguous.

**Comparison of social welfares:** The social welfare with debt is higher than the social surplus without debt if and only if:

$$\frac{23\bar{\alpha}^2 + 60\bar{\alpha}\alpha_H - 24\alpha_H^2}{242} > \frac{59}{242}\bar{\alpha}^2 \Leftrightarrow 0 > 3\bar{\alpha}^2 - 5\bar{\alpha}\alpha_H + 2\alpha_H^2 \Leftrightarrow \frac{2}{3}\alpha_H < \bar{\alpha} < \alpha_H \Leftrightarrow \alpha_L > \frac{\mu - \frac{1}{3}}{\mu}\alpha_H$$

Social welfare increases if the likelihood of low demand is low or if  $\alpha_H$  is not too different from  $\alpha_L$ . In debt-free equilibrium, the marginal cost of the private firm is lower than the expected price. It is therefore socially desirable to increase the output of the private firm. In addition, in the equilibrium without debt, the private firm has a lower marginal cost than the state-owned firm. It is therefore possible to reduce total costs by moving part of the production from the public firm to the private firm. The indebtedness of the private firm encourages it to focus only on the state of nature where demand is high. This encourages the private firm to increase its production and the state-owned firm to reduce its production. These movements increase the total production and reduce the production of the public company. These two movements contribute to an increase in the social surplus provided that they are not excessive. If  $\alpha_H$  is significantly higher than  $\alpha_L$  and if  $\mu$  is high, the debt could lead the private firm to choose a level of production such that its marginal cost would be higher than the expectation of the equilibrium price. In this case, the output of the private firm would be excessive and the social welfare could be lower than in the absence of debt. But while the increase in the private firm's production remains moderate, the debt of the private firm causes an increase in the social surplus. The social welfare increases when the production of the private firm increases little (this corresponds to the case where  $\alpha_H$  is close to  $\alpha_L$ ) or when the state of nature corresponding to a low demand has a low probability of occurring ( $\mu$  is low).

**Incentives for the private firm to go into debt:** We have considered the effects of an exogenous level of debt. We are now making the level of debt endogenous. The private firm initially has zero debt. However, it can go into debt with outside investors in exchange for the promise to repay  $D$  at the end of the game. The funds raised are immediately distributed to the shareholders of the private firm. Since it has been assumed that the capital markets are competitive, the funds obtained are equal to the expected value of a debt of  $D$ . The private firm therefore chooses the value of  $D$  which maximizes  $E(\pi_2) + E(VD)$ .

Indebtedness increases the total value of the firm if and only if:

$$\frac{32\bar{\alpha}\alpha_H - 9\bar{\alpha}^2 - 15\alpha_H^2}{121} > \frac{8}{121}\bar{\alpha}^2 \Leftrightarrow 0 > 17\bar{\alpha}^2 - 32\bar{\alpha}\alpha_H + 15\alpha_H^2 \Leftrightarrow \frac{15}{17}\alpha_H < \bar{\alpha} < \alpha_H \Leftrightarrow \alpha_L > \left(1 - \frac{2}{17\mu}\right)\alpha_H$$

<sup>13</sup>  $\frac{9}{121}\bar{\alpha}^2 > \left(\frac{4\bar{\alpha} - \alpha_H}{11}\right)^2$

**Proposition 1** *The financial structure chosen by the private firm in equilibrium is given by:*

*If  $\alpha_L < \frac{\mu - \frac{2}{17}}{\mu} \alpha_H$ , the private firm chooses a low level of debt:  $D < (\alpha_L - \frac{7}{11} \bar{\alpha}) \frac{2}{11} \bar{\alpha}$ .*

*If  $\alpha_L > \frac{\mu - \frac{2}{17}}{\mu} \alpha_H$ , the private firm chooses a high level of debt:  $(\alpha_L - \frac{7}{11} \bar{\alpha}) \frac{2}{11} \bar{\alpha} < D < (\alpha_H - \frac{2\bar{\alpha} + 5\alpha_H}{11}) \frac{3\alpha_H - \bar{\alpha}}{11}$ .*

The private firm selects a sufficiently large amount of debt to change its behavior during the competition stage in quantities if  $\alpha_L$  is close to  $\alpha_H$  or if  $\mu$  is low. The condition we have just obtained is more restrictive than the previous one. We conclude from this that if the private firm has an interest in using debt, the social welfare will increase.

**Proposition 2** *If the private firm chooses to go into debt in order to change its behavior during the competition stage in quantities, this causes an increase in the expected consumer surplus and expected social welfare.*

## 4 Private duopoly

In this section, we solve the model again, but assuming that both firms are private (and maximize their profit). This will allow, first, to analyse the impact of the nature of the ownership of the competing firm on the financial structure chosen by a private firm and, second (in the next section), to determine the impact on the social surplus of a privatization of the public firm.

Both firms are likely to incur significant debt in order to change their best response function. The choice of debt of one firm probably depends on that of the other firm. In order to determine the choice of indebtedness of firms, it is necessary to compare the total values of firms in the different possible cases. We begin by calculating the expected values of firms by distinguishing between the case where no firm is heavily indebted, the case where only one of the firms is highly indebted and the case where the two firms are heavily indebted.

### 4.1 Cournot competition without debt

The best response functions are identical to those of the private firm in the mixed duopoly. The quantities chosen by the firms in equilibrium are:

$$\left\{ \begin{array}{l} q_1 = \frac{1}{4} (\bar{\alpha} - q_2) \\ q_2 = \frac{1}{4} (\bar{\alpha} - q_1) \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} q_1 = \frac{1}{5} \bar{\alpha} \\ q_2 = \frac{1}{5} \bar{\alpha} \end{array} \right\}$$

The equilibrium price and payoffs for a given value of  $\alpha$  are equal to:

$$p = \alpha - \frac{2}{5} \bar{\alpha} \quad ; \quad \pi_i = \frac{1}{25} (5\alpha - 3\bar{\alpha}) \bar{\alpha} \quad ; \quad CS = \frac{2}{25} \bar{\alpha}^2 \quad ; \quad W = \frac{2}{25} (5\alpha - 2\bar{\alpha}) \bar{\alpha}$$

The expected payoffs are:

$$E(\pi_i) = \frac{2}{25}\bar{\alpha}^2 \quad ; \quad E(CS) = \frac{2}{25}\bar{\alpha}^2 \quad ; \quad E(W) = \frac{6}{25}\bar{\alpha}^2$$

## 4.2 Cournot competition when one firm is in debt

We assume that firm 2 is the firm with a high debt. Its best reply is:

$$q_2(q_1) = \frac{1}{4}(\alpha_H - q_1)$$

While the best reply of firm 1 is:

$$q_1(q_2) = \frac{1}{4}(\bar{\alpha} - q_2)$$

By solving the system composed of these two equations, we obtain the equilibrium quantities:

$$\left\{ \begin{array}{l} q_2 = \frac{1}{4}(\alpha_H - q_1) \\ q_1 = \frac{1}{4}(\bar{\alpha} - q_2) \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} q_2 = \frac{4\alpha_H - \bar{\alpha}}{15} \\ q_1 = \frac{4\bar{\alpha} - \alpha_H}{15} \end{array} \right\}$$

We deduce from these quantities the equilibrium price and the payoffs for a given value of  $\alpha$ :

$$\begin{aligned} p(\alpha) &= \alpha - \frac{\bar{\alpha} + \alpha_H}{5} \\ \pi_1(\alpha) &= \left( \alpha - \frac{7\bar{\alpha} + 2\alpha_H}{15} \right) \frac{4\bar{\alpha} - \alpha_H}{15} \\ \pi_2(\alpha) &= \begin{cases} 0 & \text{si } \alpha = \alpha_L \\ 2 \left( \frac{4\alpha_H - \bar{\alpha}}{15} \right)^2 - D_2 & \text{si } \alpha = \alpha_H \end{cases} \\ VD(\alpha) &= \begin{cases} \mu \left( \alpha_L - \frac{2\bar{\alpha} + 7\alpha_H}{15} \right) \frac{4\alpha_H - \bar{\alpha}}{15} & \text{si } \alpha = \alpha_L \\ D_2 & \text{si } \alpha = \alpha_H \end{cases} \\ CS(\alpha) &= \frac{1}{2} \left( \frac{\bar{\alpha} + \alpha_H}{5} \right)^2 \\ W(\alpha) &= \left( \alpha - \frac{7\bar{\alpha} + 2\alpha_H}{15} \right) \frac{4\bar{\alpha} - \alpha_H}{15} + \left( \alpha - \frac{2\bar{\alpha} + 7\alpha_H}{15} \right) \frac{4\alpha_H - \bar{\alpha}}{15} + \frac{1}{2} \left( \frac{\bar{\alpha} + \alpha_H}{5} \right)^2 \end{aligned}$$

And the expected payoffs:

$$\begin{aligned} E[\pi_1(\alpha)] &= 2 \left( \frac{4\bar{\alpha} - \alpha_H}{15} \right)^2 \quad ; \quad E[\pi_2(\alpha)] = (1 - \mu) \left[ 2 \left( \frac{4\alpha_H - \bar{\alpha}}{15} \right)^2 - D_2 \right] \\ E(VD) &= \mu \left( \alpha_L - \frac{2\bar{\alpha} + 7\alpha_H}{15} \right) \frac{4\alpha_H - \bar{\alpha}}{15} + (1 - \mu) D_2 \\ E(SC) &= \frac{1}{2} \left( \frac{\bar{\alpha} + \alpha_H}{5} \right)^2 \quad ; \quad E(W) = \frac{47\bar{\alpha}^2 + 104\bar{\alpha}\alpha_H - 43\alpha_H^2}{450} \end{aligned}$$

### 4.3 Cournot competition when both firms are in debt

When both firms are heavily indebted, they only take into account the state of the nature in which the level of demand is high. Their best replies are:

$$q_i(q_j) = \frac{1}{4}(\alpha_H - q_j)$$

Hence the equilibrium quantities:

$$\left\{ \begin{array}{l} q_2 = \frac{1}{4}(\alpha_H - q_1) \\ q_1 = \frac{1}{4}(\alpha_H - q_2) \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} q_2 = \frac{1}{5}\alpha_H \\ q_1 = \frac{1}{5}\alpha_H \end{array} \right\}$$

Of which the equilibrium price and the payoffs for a given  $\alpha$  are deducted:

$$\begin{aligned} p(\alpha) &= \alpha - \frac{2}{5}\alpha_H \\ \pi_i(\alpha) &= \begin{cases} 0 & \text{si } \alpha = \alpha_L \\ \frac{2}{25}\alpha_H^2 - D_i & \text{si } \alpha = \alpha_H \end{cases} \\ VD_i(\alpha) &= \begin{cases} \mu(\alpha_L - \frac{3}{5}\alpha_H)\frac{1}{5}\alpha_H + (1-\mu)D_i & \text{si } \alpha = \alpha_L \\ D_i & \text{si } \alpha = \alpha_H \end{cases} \\ CS(\alpha) &= \frac{2}{25}\alpha_H^2 \quad ; \quad W(\alpha) = 2\left(\alpha - \frac{3}{5}\alpha_H\right)\frac{1}{5}\alpha_H + \frac{2}{25}\alpha_H^2 \end{aligned}$$

As well as expected payoffs:

$$\begin{aligned} E[\pi_i(\alpha)] &= (1-\mu)\left[\frac{2}{25}\alpha_H^2 - D_i\right] \quad ; \quad E(VD_i) = \mu\left(\alpha_L - \frac{3}{5}\alpha_H\right)\frac{1}{5}\alpha_H + (1-\mu)D_i \\ E(CS) &= \frac{2}{25}\alpha_H^2 \quad ; \quad E(W) = \frac{10}{25}\bar{\alpha}\alpha_H - \frac{4}{25}\alpha_H^2 \end{aligned}$$

### 4.4 Financial structure choice

By comparing the total values of a firm obtained in the different cases above, we can determine its incentives to use a significant level of debt for strategic reasons. These incentives depend on the level of debt of the competing firm. So we start by determining the choice of a firm's level of debt for a given level of debt of its competitor. We then specify Nash's equilibria for this stage of the game.

**Choice to go into debt when the other firm is not in debt:** The expected total value of the firm by going into debt is greater than that which it obtains without going into debt if and only if:

$$\left(\frac{13\bar{\alpha} - 7\alpha_H}{15}\right)\frac{4\alpha_H - \bar{\alpha}}{15} > \frac{2}{25}\bar{\alpha}^2 \Leftrightarrow 0 > 31\bar{\alpha}^2 - 59\bar{\alpha}\alpha_H + 28\alpha_H^2 \Leftrightarrow \frac{28}{31}\alpha_H < \bar{\alpha} < \alpha_H \Leftrightarrow \alpha_L > \frac{\mu - \frac{3}{31}}{\mu}\alpha_H$$

This condition has a similar form to that obtained in the mixed duopoly. Debt is interesting if  $\alpha_L$  is close to  $\alpha_H$  or if  $\mu$  is low. It can be stressed, however, that this condition is more difficult to satisfy than that obtained in the mixed duopoly. A private firm uses debt for strategic reasons more often in a mixed duopoly than in a private duopoly.

**Choice to go into debt when the other firm is in debt:** Debt is the best response to debt if and only if:

$$\left(\bar{\alpha} - \frac{3}{5}\alpha_H\right) \frac{1}{5}\alpha_H > 2 \left(\frac{4\bar{\alpha} - \alpha_H}{15}\right)^2 \Leftrightarrow 0 > 32\bar{\alpha}^2 - 61\bar{\alpha}\alpha_H + 29\alpha_H^2 \Leftrightarrow \frac{29}{32}\alpha_H < \bar{\alpha} < \alpha_H \Leftrightarrow \alpha_L > \frac{\mu - \frac{3}{32}}{\mu}\alpha_H$$

A private firm can increase its expected profit by incurring debt (when the other firm is also in debt) if and only if:

$$\frac{29}{32}\alpha_H < \bar{\alpha} < \alpha_H$$

We rewrite the previous condition:

$$\bar{\alpha} > \frac{29}{32}\alpha_H \Leftrightarrow \alpha_L > \frac{\mu - \frac{3}{32}}{\mu}\alpha_H$$

The condition obtained is similar to the previous one, but a little more difficult to verify. There are therefore parameters values for which only one firm chooses to go into debt in equilibrium.

**Financial structures in equilibrium:** From the best responses of the firms, we obtain the financial structures chosen by the firms in equilibrium:

**Proposition 3** *If  $\alpha_L < \frac{\mu - \frac{3}{31}}{\mu}\alpha_H$ , firms choose low debt ( $D_i < \frac{1}{25}(5\alpha_L - 3\bar{\alpha})\bar{\alpha}$ ), which does not change their behavior during the quantities competition stage.*

*If  $\frac{\mu - \frac{3}{31}}{\mu}\alpha_H < \alpha_L < \frac{\mu - \frac{3}{32}}{\mu}\alpha_H$ , one firm chooses a high debt ( $\frac{1}{25}(5\alpha_L - 3\bar{\alpha})\bar{\alpha} < D_i < 2\left(\frac{4\alpha_H - \bar{\alpha}}{15}\right)^2$ ) and the other firm chooses to incur only a small amount of debt ( $D_j < \left(\alpha_L - \frac{7\bar{\alpha} + 2\alpha_H}{15}\right)\frac{4\bar{\alpha} - \alpha_H}{15}$ ).*

*If  $\alpha_L > \frac{\mu - \frac{3}{32}}{\mu}\alpha_H$ , the two firms choose a high debt ( $\left(\alpha_L - \frac{7\bar{\alpha} + 2\alpha_H}{15}\right)\frac{4\bar{\alpha} - \alpha_H}{15} < D_i < \frac{2}{25}\alpha_H^2$ ).*

**Comparison with the mixed duopoly:** We compare the thresholds at which firms use debt for strategic reasons. We have:  $\frac{\mu - \frac{3}{31}}{\mu}\alpha_H > \frac{\mu - \frac{2}{17}}{\mu}\alpha_H$ . The set of values of the parameters for which strategic debt appears in the mixed duopoly includes the one where strategic debt is used in a private duopoly. We can therefore say:

**Proposition 4** *The use of debt for strategic reasons is more common in a mixed duopoly than in a private duopoly.*

However, we can point out the limitations of this result. It was obtained with very specific assumptions. We will therefore avoid giving it too broad a scope.

This result does not imply that a private firm is more indebted if it is confronted with a public firm rather than a private one. Indeed, in the absence of debt, the profit of the private firm is lower if it is in competition with a state-owned firm than if it is in competition with another private firm. Debt therefore begins to have a strategic effect for a lower level in a mixed duopoly than in a private duopoly. The same level of debt can therefore have a strategic effect in a mixed duopoly and no effect in a private duopoly. By continuity, higher indebtedness may have no effect in a private duopoly while lower indebtedness has a strategic effect in a mixed duopoly.

## 5 Privatization

In this section, we study whether public authorities can increase the social welfare by privatizing the public firm before the game begins. To answer this question, we compare the total surpluses obtained in the mixed duopoly and in the private duopoly. We must distinguish four cases.

**Firms do not use strategic debt:** This case corresponds to the values of the parameters checking  $\alpha_L < \frac{\mu - \frac{2}{17}}{\mu} \alpha_H$ . Firms do not use debt strategically either in the mixed duopoly or in the private duopoly. The expected social welfare is higher in the mixed duopoly than in the private duopoly:

$$\frac{59}{242} \bar{\alpha}^2 \simeq 0.2438 \bar{\alpha}^2 > \frac{6}{25} \bar{\alpha}^2 = 0.24 \bar{\alpha}^2$$

This condition is always verified. Public authorities have no interest in privatizing the state-owned firm.

**Strategic debt is used in the mixed duopoly, but not in the private duopoly:** We are in this case when the parameters values are such that  $\frac{\mu - \frac{2}{17}}{\mu} \alpha_H < \alpha_L < \frac{\mu - \frac{3}{31}}{\mu} \alpha_H$ . In the mixed duopoly, the private firm strategically uses debt. On the other hand, in the private duopoly, firms do not use debt strategically.

We have seen previously that the use of strategic debt causes an increase in the social welfare. However, the debt-free social surplus is already higher in the mixed duopoly than in the private duopoly. The introduction of the possibility of strategic indebtedness increases the difference in social surpluses observed in both situations, but does not change their ranking. Privatizing the state-owned firm is not socially desirable.

**Only one firm is in debt in the private duopoly:** We find ourselves in this case when  $\frac{\mu - \frac{3}{31}}{\mu} \alpha_H < \alpha_L < \frac{\mu - \frac{3}{32}}{\mu} \alpha_H$ . In the mixed duopoly, the private firm goes into debt. In the private duopoly, only one firm is indebted enough to change its best response to the quantity produced by its competitor. Privatization



increases the social welfare if and only if:

$$\frac{47\bar{\alpha}^2 + 104\bar{\alpha}\alpha_H - 43\alpha_H^2}{450} > \frac{23\bar{\alpha}^2 + 60\bar{\alpha}\alpha_H - 24\alpha_H^2}{242} \Leftrightarrow \bar{\alpha} < 0, 25\alpha_H \Leftrightarrow \alpha_L < \frac{\mu - 0,75}{\mu}\alpha_H$$

If this last condition is checked, then we are not in the interval  $\frac{\mu - \frac{3}{31}}{\mu}\alpha_H < \alpha_L < \frac{\mu - \frac{3}{32}}{\mu}\alpha_H$ . Privatization is therefore never socially desirable for the values of parameters for which a single firm is strategically indebted in the private duopoly.

**The two firms go into debt in the private duopoly:** For parameters values checking  $\alpha_L > \frac{\mu - \frac{3}{32}}{\mu}\alpha_H$ , the two firms choose a sufficiently high level of debt to modify their best response during the competition stage in quantities in the private duopoly. In the mixed duopoly, the private firm also strategically uses its debt level. Privatization increases the social surplus if and only if:

$$\frac{10\bar{\alpha}\alpha_H - 4\alpha_H^2}{25} > \frac{23\bar{\alpha}^2 + 60\bar{\alpha}\alpha_H - 24\alpha_H^2}{242} \Leftrightarrow 0 > 575\bar{\alpha}^2 - 920\bar{\alpha}\alpha_H + 368\alpha_H^2$$

This polynomial admits a double root corresponding to  $\bar{\alpha} = \frac{4}{5}\alpha_H$  ( $\Leftrightarrow \alpha_L = \frac{\mu - \frac{1}{5}}{\mu}\alpha_H$ ), for which the social surpluses obtained in the two types of duopoly are indentic. For other values, privatization reduces the social surplus. The value of the double root is not in the range corresponding to this case. Thus, as in previous cases, the privatization of the public firm never increases the social welfare.

The same result is obtained in the four possible cases:

**Proposition 5** *The privatization of the public company always leads to a reduction in the social welfare.*

## 6 Conclusion

In this study, we looked at the effects of the debt of the private firm on the equilibrium of a mixed duopoly where firms compete in quantities. We focused on the effect of the private firm's limited liability. To isolate these effects, the effects of bankruptcy costs<sup>14</sup> have been neutralized by assuming they are zero.

If the debt of the private firm is sufficiently high, the private firm goes bankrupt when demand is low. Since the private firm is protected by a limited liability clause, it neglects the states of nature in which it goes bankrupt when it chooses its level of production. The over-weighting of the statements of the nature where the demand is high leads the firm to increase its production (compared to the case without debt). As quantities are strategic substitutes, the increase in the production of the private firm encourages the public firm to reduce its output level. Debt therefore has a strategic effect. It allows the private firm to encourage its public competitor to reduce its production. The strategic indebtedness of the private firm benefits consumers, as the total quantity increases and causes a fall in the price. The social welfare also

<sup>14</sup>Reviewed in Jacques (2021).

increases because, firstly, the debt encourages the private firm to produce more, but in the absence of debt this firm produces too little (its marginal cost is lower than the expected equilibrium price) and, secondly, it causes a reallocation of production between the two firms. However, in the absence of debt, the marginal cost of the public firm is higher than that of the private firm in equilibrium. The allocation of production between the two firms is therefore inefficient in the absence of debt. Debt reduces this inefficiency.

This study complements that of Jacques (2021). We focused on the limited liability effect, while Jacques focused on the impact of bankruptcy costs. In addition, the two studies used different time frames in their modelling. It seemed interesting to separate these two types of effects.<sup>15</sup> It seems necessary in a future study to bring the two types of effects together in the same model.

Other extensions are possible in future searches. It may seem desirable to expand the number of states of nature and move to a continuous set. However, the model is becoming technically more complex and it is no longer possible to obtain analytical formulas for the various results. Increasing the number of private firms also seems an attractive option. In our study, it is never optimal to privatise the public firm. However, in the absence of debt, privatization can increase the social surplus if the number of private firms is sufficiently large. Debt could have an impact on the number of private firms needed for privatization to be desirable. It also seems interesting to analyze the effect of debt when the public firm is partially privatized. Furthermore, if part of the capital of the public firm is held by private shareholders, the indebtedness of that firm would also have an effect on its best response to the quantity produced by the private firm, because the private shareholders present in the capital of the public company would also benefit from the limited liability clause.

It also seems promising to study the case where the private firm is a foreign firm. In this case, the public firm does not integrate the profit of the private firm into its objective function. This leads it to choose a level of production for which its marginal cost is higher than the equilibrium price in order to induce a transfer from the foreign private firm to domestic consumers. An increase in the output of the private firm due to its indebtedness could make this transfer even more attractive. At the same time, the nationality of the creditors of the private firm would have an impact. If the private firm is indebted to domestic investors, their interests should be taken into account in the objective function of the public firm. Indebtedness with local investors could therefore allow a foreign firm to change the objective of the public firm without opening its capital.

We could move even further away from the current model by giving the public firm a completely different objective function. Lott (1990) and Sappington and Sidak (2003) argued that public firms also had the objective of increasing their size, which could lead to predation behavior towards their private competitors. If we adopt this idea, it seems interesting to study to what extent the debt of the private firm increases the incentives of the public firm to predation.

Showalter (1995) showed that the effects of debt could depend on the mode of competition (quantities or

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<sup>15</sup>As Brander and Lewis (1986, 1988) had chosen to do for private oligopolies.

prices) in a private duopoly. Glazer (1994) argued that the effects of long-term debt could differ from those of short-term debt. It could be fruitful to analyze these variants in a mixed duopoly.

The study of the interactions between the financial structure and the strategy of firms in a mixed oligopoly therefore seems to call for a great deal of additional research in the future.

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