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# Advantageous Semi-Collusion Revisited: A Note

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**Abstract**

In this note we reconsider the paper of Brod and Shivakumar (1999), published in the Journal of Industrial Economics, who analyze a two-stage model in which the firms compete in two dimensions and examine the effect of semi-collusion when the non-production activity is R&D. They shed light on the fact that in the presence of spillovers, firms and consumers could be both better off, peradventure both worse off, by a semi-collusive production cartel. We are motivated to study their setup by this fascinating outcome. Trying to approach the in-depth analysis and to understand the driving forces of this result, we find however that the findings of Brod and Shivakumar (1999) are disputable. By focusing upon their calculative errors, we show how the correct solution can be obtained, furthermore, we explain why the main propositions in their paper don't hold.

*Keywords:* R&D, semi-collusion, spillover, product differentiation
*JEL classification:* D43, L13, O31

## Introduction

In a one stage game, cartels increase industry profits and exacerbate the consumer surplus. In a model where firms collude in production but compete in R&D, the cartel members may be worse off and consumers better off due to over-investment by firms eager to improve their position in the cartel. Brod and Shivakumar(1999) analyze a two-stage model and examine the effect of semi-collusion when the non-production activity is R&D. Firms choose their R&D

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effort in a first stage and output in a second stage. They shed light on the fact that in the presence of spillovers, firms and consumers could be both better off, peradventure both worse off, by a semi-collusive production cartel. We are attired by this fascinating outcome. Thereupon, we try to approach the in-depth analysis and to understand the driving forces of this result. We find however that the findings of Brod and Shivakumar (1999) are disputable. The incorrect Sub Game Perfect Equilibrium values of per-firm R&D effort, output and profit due to improper handling result in the inaccuracy of their main propositions. When the goods are sufficiently substitutable, the proposition 1 doesn't hold. In other words, there is no absolute predominance of production cartel in terms of R&D effort. Since the optimum equilibrium of cartel at the production stage could be negative in certain combination parameters (the degree of product differentiation and the level of spillovers), we find the region D depicted as "*Consumers prefer Production Cartel; firms prefer Competition*" could not always satisfy the conditions mentioned in proposition 2. In what follows, we firstly resume Brod and Shivakumar(1999)'s set-up; secondly, by focusing upon their calculative errors, we show what the correct solution can be and we explain why the main propositions in their paper doesn't hold.

## Model

Retrospect to the model of Brod and Shivakumar (1999), two firms with differentiated products engage in upstream R&D and downstream production. Each firm has the same production unit cost of $c$. This unit cost can be reduced by investing in R&D, where $x_i$ denotes the investment of firm $i$. Due to spillovers, the R&D expenditure $x_i$ not only benefits the investing firm $i$ but also gives rise to lower unit costs for the rival firm $j$. Then the effective production unit cost function is given by: $C_i = c - x_i - \beta x_j$ (for $i, j = 1, 2$ and $j \neq i$). The parameter $\beta \in [0, 1]$ measures the size of the spillover effect. R&D spending directly cut down the basic production unit costs $c$. Nevertheless, investing in R&D is costly, and the R&D cost function is represented by $\frac{1}{2}\delta x_i^2$, where $\delta > 0$. Let $u(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}b(q_1^2 + q_2^2 + 2\gamma q_1 q_2)$, for $(a, b > 0)$, where $\gamma \in [0, 1]$ is a product differentiation parameter whose inverse indicates the strength of production differentiation. The utility function generates the following inverse market demand curve: $p_i = a - b(q_i + \gamma q_j)$

There are two regimes: the one is Competition where firms compete in both the R&D and the output markets; the other one is Production Cartel where the firms compete in the R&D market but collude in output market. The superscript "C" stands for Competition and "P" signifies Production Cartel.

The game is solved by backward induction and we characterize the equilibrium outcomes of this game.

*Competition*:

The SGPE values of per-firm R&D effort, output and profit are given by:

$$\begin{aligned} x^C &= \frac{2A}{\theta}(2-\beta\gamma) \\ q^C &= \frac{\delta A}{\theta}(2-\gamma)(2+\gamma) \\ \pi^C &= \frac{\delta A^2 \Delta}{\theta^2} \end{aligned}$$

where $A = a - c$, $\theta = (2-\gamma)(2+\gamma)^2 b\delta - 2(1+\beta)(2-\beta\gamma) > 0$ and $\Delta = (2-\gamma)^2(2+\gamma)^2 b\delta - 2(2-\beta\gamma)^2 > 0$

In the paper of Brod and Shivakumar(1999) (Henceforth "BS"), the expression of $\Delta$ displayed in page 225 is however $\Delta_{BS} = (2-\gamma)^2(2+\gamma)^2 b\delta - 2(1+\beta)(2-\beta\gamma)^2 > 0$. We have $\Delta - \Delta_{BS} = 2\beta(2-\beta\gamma)^2 > 0$ that generates the underestimate of the real profit.

*Production Cartel*:

The symmetric equilibrium of R&D effort, output and profit correspond to the following solutions:

$$\begin{aligned} x^P &= \frac{A}{\Phi}(2-(1+\beta)\gamma) \\ q^P &= \frac{2\delta A}{\Phi}(1-\gamma) \\ \pi^P &= \frac{\delta A^2 \Gamma}{2\Phi^2} \end{aligned}$$

where $\Phi = \gamma + \beta^2\gamma + 4b\delta(1-\gamma^2) - 2\beta(1-\gamma) - 2$ and $\Gamma = -4 + 8b\delta + 8b\delta\gamma^3 + 4\gamma(1+\beta-2b\delta) - \gamma^2(1+2\beta+\beta^2+8b\delta)$. As mentioned in BS, the product $b\delta$ can be expressed in the same units as output, they assume $b\delta = 1$ to simplify expressions. And we find whether these two expressions($\Phi$,$\Gamma$) are positive or not depends on the combination of parameters $\gamma$ and $\beta$.

Whereas, BS consider that $\Phi_{BS} = 4(1-\gamma)(1+\gamma)^2 b\delta - (1+\beta)(2-(1+\beta)\gamma) > 0$ and $\Gamma_{BS} = 8(1-\gamma)^2 b\delta - (2-(1+\beta)\gamma) > 0$. Compared to our results, we have $\Phi - \Phi_{BS} = -4b\delta(1-\gamma^2)\gamma < 0$. It is clear that there is the underestimate on R&D effort and output. These errors due to improper handling generate the distinctive change in the following analysis. Furthermore, BS regard mistakenly $\Phi_{BS}$ and $\Gamma_{BS}$ as the positive terms. Taking $\Phi_{BS}$ as an example, we illustrate here $\Phi_{BS}$ is negative when

- $\gamma \in (0.927441, 0.927886]$ and $\beta \in (\tilde{\beta_1}, \tilde{\beta_2})$
- $\gamma \in [0.927886, 1]$ and $\beta \in (0, \tilde{\beta_2})$

with $\tilde{\beta_1} = \frac{1-\gamma}{\gamma} - \sqrt{\frac{1-4\gamma-4\gamma^2+4\gamma^3+4\gamma^4}{\gamma^2}}$ and $\tilde{\beta_2} = \frac{1-\gamma}{\gamma} + \sqrt{\frac{1-4\gamma-4\gamma^2+4\gamma^3+4\gamma^4}{\gamma^2}}$

## A reappraisal of the main propositions in BS(1999)

*Proposition 1*

Since $\Phi_{BS} > 0$, BS claimed the R&D effort in regime Production Cartel is always significant, the firms colluding in output spared no effort to invest in R&D for $0 \le \beta \le 1$ and for all $0 \le \gamma < 1$. In fact, their finding is not true, the crux of the matter is that the $\Phi$ could be negative[1]in certain circumscription where the optimum equilibrium R&D effort is meaningless. We find that the member firm of cartel could have no interest in R&D processus when the goods are sufficiently homogenous, precisely $\gamma \in (\hat{\gamma}, 1]$ with $\hat{\gamma} = \frac{(1+\beta)^2+\sqrt{33-28\beta+6\beta^2+4\beta^3+\beta^4}}{8}$. In this instance, the $x^P$ will be inferior to $x^C$, then the proposition 1 is not always true.

In addition, BS(1999) claimed that "*it is easy to show that as $\beta$ rises, the difference $x^P - x^C$ declines*" in page 226. As a matter of fact, the $\frac{\partial(x^P-x^C)}{\partial\beta}$ could be positive. Whether this gap enlarges or shrinks depends upon the combination of two parameters $\beta$ and $\gamma$. In order to be more legible and intuitionistic, we illustrate this outcome with the following graphic.

[1] $\Phi - \Phi_{BS} = -4b\delta(1-\gamma^2)\gamma < 0$.

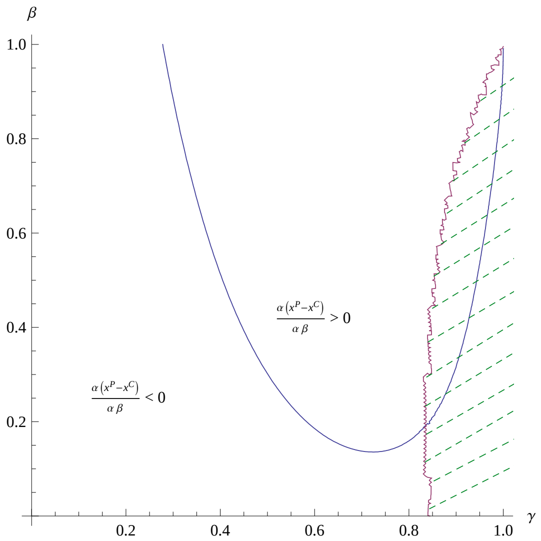


Figure 1: The effect of $\beta$ on the difference $x^P - x^C$

Apart from the dashed zone which represents the flaw of their proposition 1, we have not only the region, corresponding to the finding of BS, in which the relative valuation of R&D is reduced as spillovers increase, but also the region where the gap enlarges following the rise of spillovers. The primary reason of omitting this positive aspect of $\beta$ stems from the underestimate of R&D effort in regime P.

*Proposition 2*

BS(1999) try to compare two mentioned regimes in terms of both individual and collective incentive. They consider output as an index of consumer surplus.

$$
\begin{aligned}
q^P - q^C &= \frac{2\delta A}{\Phi}(1-\gamma) - \frac{\delta A}{\theta}(2-\gamma)(2+\gamma) \\
&= \frac{A\delta\Big(2(1-\gamma)\theta - (2-\gamma)(2+\gamma)\Phi\Big)}{\Phi\theta}
\end{aligned}
$$

It is straightforward $q^P - q^C$ has the same sign as the following expression:

$$
f(\gamma,\beta) = \frac{2(1-\gamma)\theta - (2-\gamma)(2+\gamma)\Phi}{\Phi\theta} = \frac{f_{BS}(\gamma,\beta)}{\Phi\theta}
$$

Due to improper handling and error of judgement about $\Phi$, it is mistakenly deemed that the difference $q^P - q^C$ has the same sign as the expression $f_{BS}(\gamma,\beta) = 2(1-\gamma)\theta - (2-\gamma)(2+\gamma)\Phi = -2\gamma^4 + (\beta^2+2\beta+3)\gamma^3 - 2\gamma^2(2\beta^2 + 3\beta - 3) - 4\gamma(1-\beta)$ displayed in page 227. As the case stands, the difference $q^P - q^C$ is also influenced by the denominator $\Phi\theta$.

Concerning the difference of profit $\pi^P - \pi^C$,

$$\begin{aligned}\pi^P - \pi^C &= \frac{\delta A^2\Gamma}{2\Phi^2} - \frac{\delta A^2\Delta}{\theta^2} \\ &= \frac{A^2\delta(\Gamma\theta^2 - 2\Delta\Phi^2)}{2\Phi^2\theta^2} \\ &\neq \frac{A^2\delta(\Gamma_{BS}\theta^2 - 2\Delta_{BS}\Phi_{BS}^2)}{2\Phi_{BS}^2\theta^2}\end{aligned}$$

it is straightforward that $\pi^P - \pi^C$ has the same sign as

$$g(\gamma,\beta) = \Gamma\theta^2 - 2\Delta\Phi^2 \neq \Gamma_{BS}\theta^2 - 2\Delta_{BS}\Phi_{BS}^2$$

According to Figure 2 in page 228, there are always $q_{BS}^P > q_{BS}^C$ and $\pi_{BS}^P < \pi_{BS}^C$ in region D. Practically, we can find the inverse outcome $q^P < q^C$ even $\pi^P > \pi^C$ in this region.

## References


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