

# Capital mobility, search unemployment and labor market policies: The case of minimum wages

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## Capital mobility, search unemployment and labor market policies: The case of minimum wages

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#### Abstract

In order to study the influence of capital mobility on labor market policy, this paper adapts the search-matching approach to an economy with an exogenous stock of capital. Contrary to most matching models, laissez-faire is unavoidably inefficient. However, public policy can neutralize this market failure by implementing a minimum wage. This result leads us to address a much-debated issue: Does capital mobility constrain labor market policies when governments cannot cooperate? To that end we extend the analysis to a n-country symmetric model where the setting of minimum wages results from a Nash non-cooperative game. We find that, in this context, capital mobility does not affect the efficiency of public policy.

Key words : Capital mobility, Search unemployment, Minimum wage efficiency.

JEL Classification numbers : F4, J6, H7.

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## 1 Introduction

In the same spirit as tax competition literature, the purpose of this paper is to study the influence of capital mobility on the efficiency of labor market policies when governments cannot cooperate.

For this purpose, we first adapt the search-matching approach to an economy whose capital is exogenous. This model allows us to arrive at what might thought to be unusual answers to usual questions. In particular, we find that laissez-faire is inefficient whether the Hosios rule holds or not: job creation is unavoidably excessive. However, public policy can mitigate this market failure by introducing a minimum wage that makes the decentralized equilibrium coincide with a social optimum. This result leads us to extend our analysis to multiple identical countries between which capital is perfectly mobile. As in fiscal competition models (see Wildasin, 1988, Wilson, 1985 and Zodrow and Mieszkowski, 1986 for seminal papers on this topic), we study the non-cooperative equilibrium of a game in which each government sets the minimum wage so as to maximize aggregate income, subject to the constraint of perfect capital mobility. The result we arrived at (in what we regard to be a first pass) surprised us: The Nash equilibrium of this international game is efficient.

Like most fiscal competition models, our setting is static and the capital stock of the economy is exogenous<sup>1</sup>. But; contrary to this literature, the labor market is imperfect. There are search frictions which are summarized with a CRTS matching function. Wages are bargained and firms freely enter the labor market. A specific feature of our analysis relates to job creation. The cost of creating a job is not a search cost; it relates to the (endogenous) amount of capital that firms choose to devote to each job. We assume that firms set the capital per job *ex ante*; that is, before searching for a worker. Since the output of a (filled) job is assumed to be an increasing function of the capital per job, firms decide on the capital per job by maximizing their expected profit. Under the assumption of free-entry, it turns out that the equilibrium value of the capital per job is determined by equalizing the

<sup>&</sup>lt;sup>1</sup>Since we do not deal with the issue of dynamic efficiency, we do not need to know how the capital stock of the economy is determined.

marginal productivity of labor to the value of leisure. Job creation is then obtained by dividing the global capital stock of the economy by the (privately) optimal value of the capital per head. As a consequence free-entry sets the capital price.

The manner in which the capital per job is determined explains the welfare properties of this matching model. As they are very small, firms do not internalize the fact that an increase in capital per job raises the probability of filling a vacancy by lowering market tightness. Consequently capital per job is insufficient and job creation is excessive. The externality behind this market failure is similar to the so-called congestion effect that explains why job creation is inefficient in the basic matching model (except for the non-generic case where the Hosios rule is fulfilled). In the usual model however job creation is excessive when workers' bargaining strength is weak;<sup>2</sup> whereas job creation is insufficient in the reverse case. In our setting, job creation is unavoidably excessive whatever the bargaining strengths might be. Once again, the reason for this is that capital per job does not depend on the Nash rule. It is worth noting that this means that the inefficiency of the capital per job does not stem from a holdup phenomenon (Grout, 1984). The reason for this is that the capital price is endogenous.

Following Flinn (2006), the introduction of a minimum wage is seen as means to mitigate this inefficiency by reducing job creation. However, the line of reasoning is very different. In Flinn's analysis, implementing a minimum wage acts as substitute for an increase in workers' bargaining power (which is not a policy instrument as emphasized by the author)<sup>3</sup>. Here the introduction of a minimum wage changes the manner in which firms decide on capital per job.

Turning to a long-term perspective, various papers have extended growth theory to an imperfect labor market. Pissarides (2000, Chap. 1) presents the usual search matching model as an "optimal" growth model with search unemployment. As the analysis is restricted to steady states the capital per job is determined by the modified

 $<sup>^{2}</sup>$ More precisely, job creation is too high when workers' bargaining strength is lower than the elasticity of the matching function with respect to unemployment.

 $<sup>^3 \</sup>rm One$  could object that the government can raise workers' bargaining power by using a regressive tax on wages. See Herzoug, 1984.

golden rule. Hence this study is quite different from the one presented here. Bean and Pissarides (1993), as well as Azariadis and Pissarides (2007) develop overlapping generations models with search unemployment. Contrary to the present paper, these do not deal with the issue of market efficiency. Acemoglu and Shimer (1999) also find that capital per job may be insufficient. However, in this paper where the capital price is exogenous, the inefficiency results from a holdup phenomenon.

Our analysis is closer in spirit to tax competition literature, although the perspective we adopt diverges from that which is usual for this literature. In the presence of search frictions, the pressure of capital mobility on public policies is no longer limited to the provision of public goods or to income redistribution. Search frictions often create specific externalities that justify public regulation. Our paper can be viewed as a initial exploration of the following issue: Does perfect capital mobility make labor market policies inefficient when governments do not cooperate? To begin with we show that, as regards minimum wages, such an outcome is not necessary when countries are symmetric.

To summarize, our contribution is twofold. We first state that implementing a minimum wage raises households' utility in an an economy with an exogenous capital stock (in the same way as taxes on capital improves welfare by financing the production of public goods). Second, similarly to fiscal competition models, we study the effect of capital mobility on minimum wage legislation when governments do not cooperate and we find that, in our setting, minimum wages are efficient.

Our paper is organized as follows. In the first part (sections 2 and 3) we study a closed economy that can be equated with the world economy. Section 2 outlines the search-matching model and defines a laissez-faire equilibrium. Next we state that laissez-faire is inefficient and we prove that implementing a minimum wage can make the economy equilibrium coincide with a social optimum (section 3). In the second part (section 4), we extend our analysis to a n-country model and prove that (perfect) capital mobility does not constrain the governments to set minimum wages at too low a level, even though they cannot commit to binding agreements. Section 5 concludes.

## 2 A search-matching model of a closed economy with an exogenous capital stock

We first model a closed economy that can be treated as the world economy.

#### 2.1 Market structure

The economy comprises two sets of risk neutral agents: firms and workers. Some of the firms are actively creating jobs. The labor force is homogenous and its mass is normalized to one.

As mentioned in the introduction, the capital stock of the economy is treated as an exogenous variable whose distribution among households is uniform. In other words, all workers own the same amount K of capital. Contrary to Bean and Pissarides (1993) and Azariadis and Pissarides (2007), we assume that the cost of creating a job results from the amount of capital that firms must assign to their jobs. This means that firms decide on the amount of capital per job before searching for a worker. The capital per job, k, as well as the output of a (filled) job, y, are endogenous. Because the output y is an (increasing) function f(k) of the capital k, this latter variable is an optimal choice of firms. The production function f(.) is assumed to fulfill the following usual conditions:

$$f(0) = 0,$$
  $f(\infty) = \infty,$   $f'(.) > 0,$   $f'(\infty) = 0,$   $f''(.) < 0$ 

The expected profit of a vacancy also depends on the capital price,  $\rho$ , the wage, w, that firms pay when filling their jobs and the probability q of finding a worker. In a symmetric state of the labor market the expected profit of a job, called V, then satisfies:

$$V = -\rho k + q(f(k) - w) \tag{1}$$

Following the Nash rule, the (private) surplus of a match is divided between the two parties according to their bargaining strength. Let  $\beta$  denote workers' bargaining

power  $(0 < \beta < 1)$ . We then have:

$$w = d + \beta(y - d) \tag{2}$$

Substitution of (2) into (1) yields the expected profit, V, as a function of the capital price,  $\rho$ , and the capital per job, k. We obtain:

$$V = -\rho k + q(1-\beta)(f(k)-d) \tag{3}$$

with d being the domestic output (or the utility of leisure) of unemployed workers. There are frictions in the labor market which prevents instantaneous matching of jobs with workers. These frictions are summarized with a CRTS matching function  $\dot{a}$  la Pissarides.

The probability of filling a vacancy, q, is then a decreasing function  $q(\theta)$  of the tightness of the labor market,  $\theta$ , whereas the probability of a worker finding a job, p, is an increasing function,  $p(\theta) = \theta q(\theta)$ , of market tightness. The derivative  $q'(\theta)$  is assumed to be continuous. In this discrete time setting, these probabilities also satisfy<sup>4</sup>:

$$q(0) = 1,$$
  $q(\infty) = 0,$   $p(0) = 0,$   $p(\infty) = 1$ 

Because the capital market is competitive the global capital stock, K, gives rise to the creation of jobs. Labor demand is then equal to K/k. As workers' mass is equal to one the tightness of the labor market is also given by the ratio  $\theta = K/k$ .

Like most search-matching models we retain the assumption of free-entry. We then have:

$$V = -\rho k + q(K/k)(1-\beta)(f(k)-d) = 0$$
(4)

Usually the assumption of free-entry determines job creation. Here job creation (K/k) results from firms' decisions regarding the capital per job. As a consequence, free-entry will set the capital price,  $\rho$ .

<sup>&</sup>lt;sup>4</sup>These conditions are fulfilled by the urn-ball matching function.

#### 2.2 Equilibrium

Let us first study how the capital per job, k, is set in a symmetric equilibrium. Since they are very small firms treat the probability q as an exogenous variable. Thus maximizing the expected profit gives the following first order condition:

$$-\rho + q(1 - \beta)f'(k) = 0$$
(5)

As the function f(.) is concave, the second order condition is fulfilled.

Under the assumption of free-entry (4), combining equations (3) and (5) gives:

$$\rho k = q(1-\beta)kf'(k) = q(1-\beta)(f(k)-d)$$

From the latter equalities we deduce that the capital per job, k, is determined by the following equilibrium equation:

$$f(k) - kf'(k) = d \tag{6}$$

In words the marginal productivity of employed workers should be equal to the domestic output of unemployed workers.

Consequently a laissez-faire equilibrium of the labor market can be defined as follows:

**Definition 1.** A laissez-faire equilibrium is a pair  $(k_{LF}, \rho_{LF})$  which jointly satisfies equations (4) and (6).

It is worth noting that this equilibrium is recursive. Equation (6) determines the capital per job,  $k_{LF}$ . From this equilibrium value  $k_{LF}$  one then deduces the capital price  $\rho_{LF}$  (using equation (4)) and the wage  $w_{LF}$  (using equation (2)). From  $k_{LF}$  one also obtains the tightness of the labor market  $\theta_{LF} = K/k_{LF}$  and the unemployment rate  $[1 - p(\theta_{LF})]$ .

Our model has an interesting comparative statics property. Workers' bargaining strength,  $\beta$ , does not affect the capital per job. Consequently the same holds for job creation and unemployment. An increase in workers' bargaining power only raises the wage and lowers the capital price. This property will play a major role in the

efficiency study where, contrary to most matching models, the bargaining power,  $\beta$ , is not a key parameter.

## 3 Market efficiency and public policy

We first prove that a laissez-faire equilibrium is unavoidably inefficient. Next we show that public policy can improve the performance of the labor market by introducing a minimum wage.

#### 3.1 Market efficiency

As with Hosios (1990) and many other authors, our efficiency criterion is the social surplus (also referred to as the aggregate income). Denoted by  $\sigma$ , the social surplus per head is defined as follows:

$$\sigma \equiv \sigma(k) = p(K/k)f(k) + [1 - p(K/k)]d$$
(7)

Thus a social optimum is defined as below.

**Definition 2.** A social optimum is a value of capital per job,  $k_S$ , which maximizes the social surplus ( $\sigma$ ).

This definition leads us to compute the derivative of  $\sigma$  with respect to k. Denoting the elasticity of  $q(\theta)$  in absolute values as  $\eta(\theta)$ , this derivative can be written as follows (remember that  $\theta = K/k$ ):

$$\sigma'(k) = -[1 - \eta(K/k)]p(K/k)\frac{1}{k}[f(k) - d] + p(K/k)f'(k)$$
(8)

For k > 0, this derivative has the same sign as:

$$H(k) \equiv -[1 - \eta(K/k)][f(k) - d] + kf'(k)$$

Regarding the efficiency of the labor market in the laissez-faire regime, we can state the following first proposition: **Proposition 1.** A laissez-faire equilibrium is unavoidably inefficient. **Proof.** Rearranging the terms the expression H(k) can be rewritten as:

$$H(k) = -[f(k) - kf'(k) - d] + \eta(K/k)[f(k) - d]$$

From the definition of a laissez-fair equilibrium (equation (6)) we know that:

$$f(k_{LF}) - k_{LF}f'(k_{LF}) - d = 0$$

This implies that in the neighborhood of laissez-faire the derivative of  $\sigma(.)$  with respect to k has the same sign as the private surplus:

$$y_{LF} - d > 0$$

Consequently, an  $\epsilon$ -increase in the capital per job would raise the aggregate income. This proves the proposition.

#### Q.E.D.

It is worth noting that, contrary to the basic matching model (where the Hosios rule  $(\beta = \eta)$  ensures the efficiency of the labor market), the bargaining strength of workers as well as the elasticity of the matching function do not play any role in obtaining this inefficiency outcome. The reason for this particularity is that in our setting job creation does not depend on *ex post* profits  $((1 - \beta)(y - d))$ . Here the division of the private surplus between the two parties only affects the capital price.

However, this result fails to specify the location of the laissez-faire value of k relative to its socially optimal value. To that end we first need to state the proposition below.

**Proposition 2**. The economy has a social optimum,  $k_S$ , which satisfies the first order condition: H(k) = 0.

**Proof.** For  $k = f^{-1}(d) > 0$ , the social surplus  $\sigma(k)$  (equation (7)) is equal to dand its derivative  $\sigma'(k) (= p(K/k)f'(k))$  is strictly positive. Since  $\sigma(k) < d$  for  $0 < k < f^{-1}(d)$  and  $\sigma'(.)$  is continuous, this implies that  $k_S > f^{-1}(d)$  and that  $\sigma(k_S) > d + \epsilon$ , with  $\epsilon$  being a (strictly) positive scalar. On the other hand, for  $k \to \infty$ ,  $q(K/k) \to 1$  and, since  $f'(k) \to 0$ ,  $[(f(k) - d)/k] \to 0$ . Consequently  $\sigma(k)$  tends to d and  $k_S$  is bounded. This proves the proposition. Q.E.D.

Notice that Proposition 2 only ensures the existence of a social optimum. In the following, for the sake of simplicity, the equation H(k) = 0 will be assumed to have a single solution. Thus, the social optimum  $k_s$  is unique.

Now regarding the location of  $k_{LF}$  with respect to  $k_S$ , we obtain the following result:

**Proposition 3**. Relative to a social optimum, capital per job is too low in a laissezfaire equilibrium.

**Proof**. See Appendix A.

Although our model is quite different from the basic search-matching setting, the intuition behind Proposition 4 is similar to the so-called congestion effect. Since they are very small, firms are not aware that an increase in the capital per job reduces the tightness of the labor market, which then leads to an increase in the probability (q) of filling jobs. As a consequence, the capital per job being insufficient, the congestion (i.e. the tightness) of the labor market is excessive.

It is worth noting that, contrary to Acemoglu and Shimer (1999) in which the capital price is exogenous, this inefficiency does not stem from a holdup phenomenon (Grout, 1984). In order to get a more precise understanding of this point, let us assume that the firms take into account the effect on job creation when they decide on the capital per job. In this case, the first order condition can be written as:

$$-\rho k + (1 - \beta)q(K/k)[kf'(k) + \eta(K/k)(f(k) - d)] = 0$$

Using the free-entry condition (equation (4)), we obtain:

$$H(k) = 0 \qquad \Leftrightarrow \qquad k = k_S$$

This clearly states that the inefficiency of the capital per job exclusively comes from the fact that firms do not internalize its effect on job creation.

#### 3.2 Public policy: introducing an optimal minimum wage

We have seen that laissez-faire is inefficient. Can public policy mitigate this market failure?

In what follows we prove that introducing a minimum wage can restore the efficiency of the labor market under one condition. Workers' bargaining strength,  $\beta$ , must be lower than the (socially) optimal value of the elasticity  $\eta(.)$ .

We first have to define a minimum-wage equilibrium. To that end that, let us assume that the government sets a *binding* minimum wage, m. In other words the minimum wage m is greater than the bargained wage:

$$m \ge d + \beta(y - d) \tag{9}$$

Under this binding condition, firms will decide on the capital per job, k, by maximizing the expected profit V (equation (1)) for w = m. Thus the variable k must satisfy the first order condition:

$$\rho = q(K/k)f'(k) \tag{10}$$

Remember that firms treat  $\theta(=K/k)$  as an exogenous variable.

Under the assumption of free-entry, substitution of the latter equation into (1) yields:

$$f(k) - kf'(k) = m \tag{11}$$

In words, labor marginal productivity becomes equal to the (minimum) wage.

As a consequence, a minimum wage equilibrium of the labor market can be defined as follows:

**Definition 3.** A minimum wage equilibrium is a pair (m, k) which jointly satisfies the binding condition (9) and the capital per job equation (11).

Since the marginal productivity of labor is an increasing function of the capital per job, this suggests that the minimum wage could be set in such a way that firms choose the (socially) optimal value of capital per job. This optimal minimum wage, denoted by  $m_S$ , should be determined by:

$$m_S = y_S - k_S f'(k_S)$$

However things are not that simple. This is because this level of the minimum wage is not necessarily binding. Indeed, as a social optimum satisfies:

$$y_S - k_S f'(k_S) = d + \eta(\theta_S)(y_S - d)$$

we must have:

$$m_S = d + \eta(\theta_S)(y_S - d) \ge d + \beta(y_S - d)$$

This means that restoring the efficiency of the economy with a minimum wage requires that the bargaining strength,  $\beta$ , be lower than the optimal value of the elasticity  $\eta(.)$ , denoted by  $\eta_s$ .

The following proposition summarizes the previous results.

**Proposition 4.** Under the condition  $\beta \leq \eta_S$ , the minimum wage  $m_S$  makes the equilibrium coincide with a social optimum.

Assuming that the elasticity  $\eta$  is a constant, different empirical papers find that the bargaining strength  $\beta$  is smaller than  $\eta$ . So the condition  $\beta \leq \eta$  sounds plausible.

To conclude this part, we would like to add that maximizing the social surplus is all the more relevant since this criterion coincides with the expected utility of a household, whether wage setting results from Nash bargaining or from the law. Implementing the minimum wage  $m_S$  is a Pareto-improving measure.

# 4 Does capital mobility constrain minimum wage policy?

A much-debated issue is the extent to which capital mobility constrains labor market public policies. According to many economists and politicians, in a non-cooperative context capital mobility would constrain governments to weaken labor market regulation. In particular, they should set lower minimum wages in order to avoid a capital drain that would lead to an increase in unemployment (as established above). In our setting, such a constraint would be all the more prejudicial since minimum wages are used as a means of neutralizing market failure. To address this major issue, we now extend the analysis to a world economy that is divided into multiple identical countries between which capital mobility is perfect. In the same vein as fiscal competition models (see Wildasin (1988) for a seminal full-employment model), in the non-cooperative situation governments are assumed to play a Nash game whose strategies are in our case the minimum wages. The Nash equilibrium is then compared with the cooperative equilibrium. As countries are identical, the cooperative equilibrium of the n-country model coincides with the social optimum of the one-country model (Definition 2). The result we arrive at sounds surprising: the non-cooperative equilibrium is efficient. In other words, (perfect) capital mobility does not exert any influence on minimum wage legislation.

We first expose this Nash game.

#### 4.1 Minimum wage setting as a Nash game.

The world economy is divided into  $n \ (n > 1)$  identical open countries  $i \ (i = 1, 2, ..., n)$ . All countries i have the same structure as the closed economy we studied above. For expositional simplicity the workers' mass in each country is normalized to one. As a consequence, each household owns a capital stock of amount K (in a fact it is a density).

As in Definition 3, a binding minimum wage  $m_i$   $(m_i \ge d + \beta(y_i - d))$  determines the capital per job in country *i*. We still have  $\forall i \in \{1...n\}$ :

$$f(k_i) - k_i f'(k_i) = m_i$$

Since the minimum wages have no other direct effect, we can likewise formulate the Nash game in terms of the capitals per job  $k_i$  (i = 1, 2, ..., n).

In the following we compute a (symmetric) non-cooperative equilibrium under the assumption that the minimum wages are binding. We will then check that the equilibrium value of the minimum wage (equal to labor marginal productivity) actually is higher than the equilibrium value of the bargained wage.

#### 4.1.1 Minimum wages and capital flows.

On the other hand, perfect mobility implies that capital has the same price,  $\rho$ , in all countries. From equation (10) we deduce:

$$\rho = q(K_i/k_i)f'(k_i)$$

with  $K_i$  being the capital invested in country i (i = 1, 2, ..., n). We then have:

$$q(K_1/k_1)f'(k_1) = \dots = q(K_i/k_i)f'(k_i) = \dots = q(K_n/k_n)f'(k_n)$$
(12)

Knowing that

$$\sum_{i=1}^{n} K_i = nK$$

the law of unique price (equation (12)) determines the capital stock  $K_i$  as an implicit function of the capital per job  $k_i$ :

$$K_i \equiv \kappa(k_i, .)$$

where the point denotes the capital per job in the other countries  $(j \neq i)$ . Let  $\phi(.)$  denote the elasticity of the marginal productivity of capital f'(.):

$$\phi(k) \equiv \frac{f''(k)k}{f'(k)} < 0$$

Differentiating (12) gives the derivative of the function  $\kappa(k_i, .)$  with respect to  $k_i$ . In a symmetric state of the world economy, we obtain<sup>5</sup>:

$$\frac{\partial K_i}{\partial k_i} = \frac{n-1}{n\eta(K/k)} \frac{K}{k} [\phi(k) + \eta(K/k)]$$
(13)

 $<sup>^5\</sup>mathrm{See}$  Appendix A for detailed calculus.

It is worth noting that an increase in capital per job  $k_i$  (resulting from an increase in the minimum wage  $m_i$ ) does not necessarily generate a capital outflow. The reason for this is that, holding  $K_i$  as a constant, such a minimum wage increase has two opposite effects on the return to capital in country i. On the one hand it tends to lead to an outflow by lowering the marginal productivity of capital in country i (first term in the brackets). But in the presence of search frictions, the decrease in market tightness raises the probability of filling a job  $q(K_i/k_i)$ . This tends to make country i more attractive to investors and can lead to a capital inflow (second term in the brackets).

#### 4.1.2 Governments' payoff and non-cooperative international equilibrium.

The payoff functions of this game are deduced from social surpluses. Ex ante, in this n-country model, capital investments  $K_i$  no longer coincide with capital endowments K. Consequently social surpluses cannot be defined as above. For the sake of clarity, we first define the social surplus of country i,  $\sigma_i$ , as the sum of its inhabitants' incomes, whether these incomes are paid by residential or non-residential firms. We obtain:

$$\sigma_i = K\rho + p(K_i/k_i)m_i + [1 - p(K_i/k_i)]d$$

One can see that the social surplus can be rewritten as:

$$\sigma_i = (K - K_i)q(K_i/k_i)f'(k_i) + p(K_i/k_i)f(k_i) + [1 - p(K_i/k_i)]d$$

Substitution of the implicit function  $\kappa(k_i, .)$  into  $\sigma_i$  yields the payoff functions of the non-cooperative game:

$$\sigma(k_i, .) = (K - \kappa(k_i, .))q(\kappa(k_i, .)/k_i)f'(k_i) + p(\kappa(k_i, .)/k_i)f(k_i) + [1 - p(\kappa(k_i, .)/k_i)]d$$
(14)

In order to characterize a non-cooperative equilibrium we are led to compute the derivative of the payoff  $\sigma(k_i, .)$  with respect to  $k_i$  in a symmetric state of the world

economy  $(k_i = k, \forall i = 1...n)$ . As  $K_i = K, \forall i \in \{1, 2, ..., n\}$ , setting this derivative to zero gives<sup>6</sup>:

$$-[\eta(K/k) - (n-1)\phi(k)][f(k) - kf'(k) - d - \eta(K/k)(f(k) - d)] = 0$$

Since  $\phi(.) < 0$ , we obtain:

$$f(k) - kf'(k) - [d + \eta(K/k)(f(k) - d)] = 0 \quad \Leftrightarrow \quad H(k) = 0$$

From Propositions 2 and 4, we deduce that:

**Proposition 5.** Assuming that workers' bargaining power  $\beta$  is lower than the elasticity  $\eta_S$ , the (symmetric) non-cooperative equilibrium of the minimum wage game is efficient.

Although it is obviously driven by the assumption that countries are identical, this proposition is surprising<sup>7</sup>. One could surmise that, in the absence of cooperation, governments would set minimum wages too low because they fear a capital outflow. To shed some light on this result, it is worth presenting a less general but (much) more insightful proof of Proposition 5. To that end let us consider the aggregate income of country i as a function of  $k_i$  and  $K_i$ :

$$\sigma_i = \sigma(k_i, K_i)$$

One can check that in a symmetric state of the n-country model:

$$\frac{\partial \sigma(k_i, K_i)}{\partial K_i} = -\frac{k}{K} \frac{\partial \sigma(k_i, K_i)}{\partial k_i}$$

We then obtain:

$$\frac{\partial \sigma(k_i, .)}{\partial k_i} = \frac{\partial \sigma(k_i, K_i)}{\partial k_i} \left[1 - \frac{k}{K} \frac{\partial K_i}{\partial k_i}\right]$$

<sup>&</sup>lt;sup>6</sup>See Appendix A for detailed calculus.

 $<sup>^{7}\</sup>mathrm{In}$  general, the assumption that players are identical does not ensure that the non-cooperative equilibrium is efficient

Assuming that:

$$1 - \frac{k}{K} \frac{\partial K_i}{\partial k_i} \neq 0$$

it turns out that, in a symmetric state of the n-country model  $(K_i = K, \forall i = 1...n)$ , the governments behave as if their criterion reduced to:

$$\sigma_i = p(K_i/k_i)f(k_i) + [1 - p(K_i/k_i)]d$$

In other words, they decide on the minimum wage in the same way as the government of a closed economy (see equation (7)). In this symmetric setting, such behavior is clearly efficient.

## 5 Conclusion

In this paper, we have extended the search matching model to an economy with an exogenous stock of capital. Interestingly we found that laissez-faire is inefficient whatever workers' bargaining strength might be. In that regard we also stated that a minimum wage can restore market efficiency.

We then generalized the analysis to a n-country model and found that (perfect) capital mobility does not constrain the governments to decide on inefficient minimum wages. Different lines for further research can be considered. Although the symmetric case has some practical relevance (when considering countries with similar levels of development such as the individual states of the USA or of the Euro area), there are pressing reasons to extend the analysis to the asymmetric case. This would be especially helpful in interpreting the efficiency of non-cooperative equilibrium in the symmetric setup. Next, similar to Pissarides (2000, Chap.1), the cost of creating a job was related to the capital that firms must acquire before searching for workers. This assumption sounds plausible, but it is not standard. Even when using an overlapping generations (OLG) setting, Bean and Pissarides (1993) as well as Azariadis and Pissarides (2007) assume that the cost of creating a job is a consequence of search effort on the part of the firm. Retaining this more usual assumption would allow us to test the robustness of our results. Finally, our model should be reformulated in an OLG structure. The welfare analysis of the autarchic country resembles a static efficiency study of an OLG model with search unemployment. This must be clarified. Moreover, an OLG setting would explain how real capital can be mobile.

From a broader perspective, it would be interesting to study how capital mobility affects the implementation of other public policies that improve market efficiency in the presence of search unemployment (see for instance Blanchard and Tirole (2008) concerning unemployment benefits and layoff taxes or Gavrel (2011) regarding subsidies to non-participants).

#### 6 References

Acemoglu, D., Shimer, R., 1999. Holdups and efficiency with search frictions. International Economic Review, 40, 827-849.

Azariadis, C., Pissarides, C.A., 2007. Unemployment dynamics with international capital mobility. European Economic Review, 51, 27-48.

Bean, C., Pissarides, C.A., 1993. Unemployment, consumption and growth. European Economic Review, 37, 837-854.

Blanchard, O.J., Tirole, J., 2008. The joint design of unemployment insurance and employment protection: A first pass. Journal of the European Economic Association, 6, 45-77.

Flinn, C., 2006. Minimum wage effects on labor market outcomes under search, matching, and endogenous contact rates. Econometrica, 74, 1013-1062.

Gavrel, F., 2011. On the efficiency of participation with vertically differentiated workers. Economics Letters, 112, 100-102.

Grout, P., 1984. Investment and wage in the absence of binding contracts: A Nash Bargaining Approach. Econometrica, 52, 449-460.

Herzoug, T., 1984. Union responses to tax changes. Oxford Economic Papers, 36, 37-51.

Pissarides, C.A., 2000. *Equilibrium Unemployment Theory*, 2nd Edition, The MIT Press.

Wildasin, D.E., 1988. Nash equilibria in models of fiscal competition. Journal of Public Economics, 35, 229-240.

Wilson, J.D., 1985. Optimal property taxation in the presence of interregional capital mobility. Journal of Urban Economics, 17, 73-89.

Zodrow, G.R., Mieszkowski, P., 1986. Pigou, Tiebout, property taxation, and the underprovision of local public goods. Journal of Urban Economics, 19, 356-370.

## **Appendix A: Proof of Proposition 4**

To compare a social optimum with a laissez-faire equilibrium let us consider the parameterized equation below.

$$f(k) - kf'(k) = d + P$$
 (15)

where P is a positive parameter which lies in the interval  $[0, P_S]$ . The bound  $P_S$  is obtained by computing the expression:

$$\eta(\theta)[f(k) - d]$$

with the optimal value  $k_s$ .

The previous equation determines k as an implicit function k(P) of the parameter P. For P = 0, the solution is a laissez-faire equilibrium whereas, for  $P = P_S$ , the solution is a social optimum.

Differentiating (15) gives the derivative of the function k(P):

$$k'(P) = -\frac{1}{kf''(k)} > 0$$

Since  $P_S > 0$ , this proves that the (socially) optimal value of the capital per job is greater than its laissez-faire value. *Q.E.D.* 

## Appendix B: Detailed calculus

**Derivative of function**  $\kappa(k_i, .)$ 

In a symmetric state of the world economy, differentiating equation (12) gives (for all  $j \neq i$ ):

$$q(K/k)f''(k) + q'(K/k)f'(k)\left[-\frac{K}{k^2} + \frac{1}{k}\frac{\partial K_i}{\partial k_i}\right] = q'(K/k)f'(k)\frac{1}{k}\frac{\partial K_j}{\partial k_i}$$

On the other hand, we have:

$$(n-1)\frac{\partial K_j}{\partial k_i} + \frac{\partial K_i}{\partial k_i} = 0$$

Combining the two previous equations yields:

$$q'(K/k)f'(k)\frac{1}{k}\frac{n}{n-1}\frac{\partial K_i}{\partial k_i} = -q(K/k)f''(k) + q'(K/k)f'(k)\frac{K}{k^2}$$

Or:

$$f'(k)\frac{n}{n-1}\frac{\partial K_i}{\partial k_i} = -\frac{q(K/k)k}{q'(K/k)}f''(k) + f'(k)\frac{K}{k}$$

As

$$\frac{q(K/k)k}{q'(K/k)} = -\frac{K}{\eta(K/k)}$$

we finally obtain:

$$\frac{\partial K_i}{\partial k_i} \equiv \frac{\partial \kappa(k_i,.)}{\partial k_i} = \frac{n-1}{n} \left[\frac{K}{k} + \frac{K}{k} \frac{\phi(k)}{\eta(K/k)}\right]$$

## Derivative of function $\sigma(k_i, .)$

Differentiating equation (14) in a symmetric state of the n-country model gives<sup>8</sup>:

$$\frac{k}{q(K/k)}\frac{\partial\sigma(k_i,.)}{\partial k_i} = \left[-k\frac{\partial K_i}{\partial k_i} + K\right]f'(k) + \left[1 - \eta(K/k)\right]\left[f(k) - d\right]\left(\frac{\partial K_i}{\partial k_i} - \frac{K}{k}\right)$$

Substitution of  $\partial K_i/\partial k_i$  into the previous equation yields:

$$\frac{n\eta(.)k^2}{Kq(.)}\frac{\partial\sigma(k_i,.)}{\partial k_i} = \eta(.)kf'(k) - (n-1)\phi(.)kf'(k) + (n-1)\phi(.)(1-\eta(.))(f(k)-d) - \eta(.)(1-\eta(.))(f(k)-d) - \eta(.)(1-\eta(.))(f(k)-\eta(.$$

Or:

$$\frac{n\eta(.)k^2}{Kq(.)}\frac{\partial\sigma(k_i,.)}{\partial k_i} = -[\eta(.) - (n-1)\phi(.)][f(k) - kf'(k) - d - \eta(.)(f(k) - d)]$$

<sup>8</sup>Notice that  $K_i = K$  for all i = 1, ..., n.

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